Perfect function transfer and interference effects in interacting boson lattices

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We show how to perfectly transfer, without state initialization and remote collaboration, arbitrary functions in interacting boson lattices. We describe a possible implementation of state transfer through bosonic atoms trapped in optical lattices or polaritons in on-chip coupled cavities. Significantly, a family of Hamiltonians, both linear and nonlinear, is found which are related to the Bose-Hubbard model and that enable the perfect transfer of arbitrary functions. It is shown that the state transfer between two sites in two-dimensional lattices can result in quantum interference due to the different numbers of intermediate sites in different paths. The signature factor in nuclear physics can be useful to characterize this quantum interference.

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I. INTRODUCTION

Quantum information processing (QIP) often needs to transfer a quantum state from one site to another [1]. For example, in optical quantum communications, one may directly transmit flying photons. However, in many other tasks, e.g., solid-state-based quantum computation, quantum state transmission is not a trivial task. Completing such a task is often needed in QIP, e.g., the quantum information exchange between two separate processors. Therefore, it is very important to find physical systems that provide robust quantum state transmission lines linking different QIP processors.

In recent years, extensive investigations have been done on quantum state transfer. In particular, many results have been obtained on qubit-state transfer through spin chains with various types of neighbor couplings. The original idea of quantum state transfer through a system of interacting spins-1/2 was introduced by Bose [2]. Afterward, Chirstandl *et al.* [3] and independently Nikolopoulos *et al.* [4] found that perfect state transfer (PST) is possible [5] in spin-1/2 networks without any additional actions from senders and receivers, including not requiring switching on and off qubit couplings. These results have triggered much interest in demonstrating and optimizing reliable state transfer in different models of interacting spin chains (see e.g., [6, 7, 8] and references therein). For example, Di Franco *et al.* [9] clarified that PST can be realized in spin-1/2 chains with neither state initialization of the medium nor fine-tuned control pulses. Recently, this scheme [9] was further simplified [10] by not using remote collaboration of senders and receivers. Thus, the PST of qubit states only requires access to two spins at each end of the spin-1/2 chain.

Here, we show how to achieve PST of *any function* through bosonic lattices. Earlier schemes often assume that all the spins (besides the edge spins) are in the ground state. Here, we do not assume this.

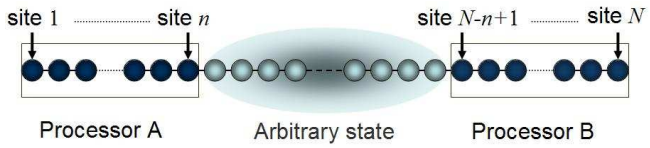

FIG. 1: (Color online) Our protocol enables a *perfect mirror transfer* of any n-variable function f from processor A to processor B through a bosonic chain initially in an *arbitrary* state.

So far, only a few works [11, 12, 13, 14] investigated the perfect transmission of an arbitrary *continuous*-variable quantum state. As it is well known, many important quantum states belong to this class, e.g., squeezed and coherent states. Also it is possible to make QIP with such states for many important tasks. Here, we propose a protocol to perfectly transfer an unknown n-variable function from a processor at one end of a boson chain to another processor at the other end (see Fig. 1). In the former schemes, *exponential*-type states (e.g., coherent or squeezed states) cannot be perfectly transferred through spin networks. It is important to emphasize that our proposal *can* do this. By using continuous-variable states, our protocol can be designed directly, since there is a one-to-one correspondence between continuous variables and bosonic operators.

The interaction required for perfect state transfer in our model can be implemented using the Bose-Hubbard model—a paradigm for studying correlated bosonic systems, such as optical lattices, Josephson junction arrays, topological excitations, and so on (see, e.g., [15] and references therein).

We stress that there is a family of Hamiltonians that can

perform the same tasks. We derive this family via the so-called *dressing* transformations, which will be described below. Moreover, we show that the state transfer between two sites in two-dimensional lattices can produce quantum interference that can be characterized by the signature factor used in nuclear theory spectroscopy [16].

II. BOSE-HUBBARD MODEL

Let us now consider dynamics of bosons, in a system with N sites, governed by the Bose-Hubbard Hamiltonian [15, 17, 18, 19]:

$$H = -\sum_{k=1}^{N-1} J_k(b_k^\dagger b_{k+1} + b_{k+1}^\dagger b_k) + \sum_{k=1}^{N} \epsilon_k n_k + H_U, \quad (1)$$

where $n_k = b_k^\dagger b_k$ is the number operator for the bosons located at the kth site, $b_k^\dagger$ (b_k) is the bosonic creation (annihilation) operator. For simplicity, we set $\hbar = 1$ and, hereafter, we drop the lower and the upper bounds of summation. Equation (1) describes hopping bosons in the presence of the on-site repulsion, given by

$$H_U = U \sum n_k(n_k - 1). \quad (2)$$

The hopping (tunneling) matrix element between nearest-neighbor sites is given by

$$J_k = \int d^3\mathbf{r}\, w^*(\mathbf{r} - \mathbf{r}_k)[T + V_{\text{lat}}(\mathbf{r})]w(\mathbf{r} - \mathbf{r}_{k+1}), \quad (3)$$

where $w(\mathbf{r} - \mathbf{r}_k)$ is a single-atom Wannier function at lattice site k, $V_{\text{lat}}(\mathbf{r})$ denotes the optical lattice potential, and T is the kinetic energy of a single atom. The parameter

$$\epsilon_k = \int d^3\mathbf{r}\, V_T(\mathbf{r})|w(\mathbf{r} - \mathbf{r}_k)|^2 \approx V_T(\mathbf{r}_k), \quad (4)$$

where $V_T(\mathbf{r})$ characterizes an additional external trapping potential. The parameters in an optical lattice are controllable. For instance, the well depth, $V_0 = \max|V_{\text{lat}}(\mathbf{r})|$, of the optical lattice can be tuned in real time by changing the power of the lasers. The parameters $V_T(\mathbf{r}_k)$ and V_0 can be controlled to obtain desired values of ϵ_k and J_k.

The model described by Eq. (1) can, e.g., be implemented in a system of interacting polaritons in periodic arrays of coupled optical resonators with strong atom-photon coupling [20] (for a review see [18]). Large-scale ($N > 100$) arrays of ultrahigh-finesse ($Q \sim 1 \times 10^6$) coupled photonic-crystal nanocavities have been realized experimentally [21] demonstrating a relatively long photon lifetime (~ 1 ns) with low propagation and coupling losses. Also ultrahigh-finesse toroidal fiber-coupled microresonators have been proposed [20], for which the observed coupling between fundamental modes of a fiber and a microresonator much exceeds losses into other modes: specifically, the experimental ideality factor was reported to be 99.97% [22]. Alternative implementations of Eq. (1) include: arrays of Josephson junctions (e.g., [23, 24]) and ultracold atoms trapped in optical lattices (as suggested in [17] and first experimentally realized in [25]). For a review on the Bose-Hubbard model and its physical implementations, see [15]. We note that, in the latter systems, it is more difficult, in comparison to coupled arrays of cavities, to access and to control individual lattice sites due to the small distances between the sites [18].

We note that when the nonlinear term H_U in Eq. (1) is negligibly small, Eq. (1) becomes a linearly coupled bosonic Hamiltonian, which is equivalent to that of the on-chip coupled cavities (e.g., as theoretically described in Refs. [20], [26], and [27]). The controllable parameter ϵ_k of these cavities can be realized by, e.g., tunable transmission line resonators (e.g., Refs. [28] and [29]), while the tunable coupling J_k between each pair of resonators can be realized by superconducting quantum interference device couplers. Recent progress in manufacturing ultrahigh-finesse microcavities on a chip [30, 31, 32, 33] indicates the experimental feasibility of state transfers in such systems with current or soon-to-be-available technology.

III. ANGULAR MOMENTUM AND ENGINEERED BOSE-HUBBARD MODEL

The angular momentum vector $\mathbf{L}$ is a single-particle operator that can have either a fermionic or a bosonic representation. The bosonic representation of $\mathbf{L}$ can be written [34] as

$$\mathbf{L} = \sum_{m,m'} \langle lm|\mathbf{L}|lm'\rangle A_{lm}^\dagger A_{lm'}, \quad (5)$$

where $|lm\rangle$ are eigenstates of the total angular momentum $\mathbf{L}^2$ and L_z. Here, we define bosonic creation operators $A_{lm}^\dagger$ such that, under rotation, they map or transfer among themselves as a rank-l irreducible spherical tensor operator.

Reference [3] maps the indices of the site numbers of a one-dimensional chain into the magnetic quantum number m of the total angular momentum l, such that $l = \frac{N-1}{2}$ and $m = -\frac{N-1}{2} + k - 1$. For instance, the magnetic number for the first site is $m = -l = -\frac{N-1}{2}$. With this mapping, the rank l irreducible spherical tensor bosonic operator $A_{lm}^\dagger$ corresponds to the bosonic operator $b_k^\dagger$ at site $k = m + \frac{N+1}{2}$, which obeys the tensor transformation [34],

$$R(\Omega)A_{lm}^\dagger R^{-1}(\Omega) = \sum_{m'} D_{m'm}^l(\Omega)A_{lm'}^\dagger, \quad (6)$$

where $R(\Omega)$ is a rotation operator with Euler angles $\Omega = (\alpha, \beta, \gamma)$, and

$$D_{m'm}^l(\Omega) = \exp(-im'\alpha)d_{m'm}^l(\beta)\exp(-im\gamma) \quad (7)$$

given in terms of Wigner's d-matrix [35], $d_{m'm}^l(\beta)$. The three components of the angular momentum vector may be

expressed by atomic creation and annihilation operators as

$$\begin{aligned} L_x &= \sum C_k(b_k^\dagger b_{k+1} + b_{k+1}^\dagger b_k), \\ L_y &= i\sum C_k(b_k^\dagger b_{k+1} - b_{k+1}^\dagger b_k), \\ L_z &= \sum n_k \left[k - \tfrac{1}{2}(N+1)\right], \end{aligned} \tag{8}$$

where $C_k = \frac{1}{2}\sqrt{k(N-k)}$. The term $\sum n_k$ in L_z is the total boson number and commutes with other operators. If we engineer J_k and ϵ_k in Eq. (1) such that [3] $J_k = JC_k$ and $\epsilon_k = \epsilon\left(\frac{N+1}{2} - k\right)$, then its time evolution becomes

$$U(t) = \exp[i(JL_x + \epsilon L_z - H_U)t]. \tag{9}$$

The on-site repulsion H_U can be adjusted by varying the strength U.

IV. LINEAR CASE

Let us first consider the case when J is so large that we can ignore H_U, so the Bose-Hubbard model becomes linear and the evolution operator $U(t)$ corresponds to a rotation operator $R(\Omega)$. Additionally, by assuming that $\epsilon = 0$, the evolution is described by

$$U^\dagger(t) A_{lm}^\dagger U(t) = \sum e^{i(\pi/2)(m'-m)} d^l_{m'm}(Jt) A^\dagger_{lm'} . \tag{10}$$

When $t_0 = \pi/J$, this expression reduces to a simple form:

$$U^\dagger(t_0)\; b_i^\dagger\; U(t_0) = r\; b^\dagger_{N-i+1}\,, \tag{11}$$

where the factor

$$r = \exp\left(-i\pi\frac{N-1}{2}\right) \tag{12}$$

is analogous to the so-called *signature* in nuclear structure theory [16]. Below, we will discuss the interference induced by this signature factor. There exist observable effects for different signatures in nuclear spectroscopy. For instance, the lowest (so-called bandhead [16]) energy differs for the two cases $r = -i$ and $r = i$, for the same spin $K = 1/2$. Here, $r = 1$ when the total number of sites $N = 5, 9, 13, \ldots$. In this case

$$U^\dagger(t_0)\; b_i^\dagger\; U(t_0) = b^\dagger_{N-i+1}. \tag{13}$$

Moreover, these results are valid for a general linear Hamiltonian $H_l = \mathbf{v}\cdot\mathbf{L}$, with constant vector $\mathbf{v} = (-J, 0, -\epsilon)$.

V. PERFECT STATE TRANSFER FOR THE LINEAR CASE

The following results can be applied to various linear optical or atomic systems. Now, let us assume that a multivariable function f is encoded in an n-site processor A, such that $f(x_1, ..., x_n)$ is mapped into the state $f(b_1^\dagger, ..., b_n^\dagger)|\mathbf{0}\rangle$ as follows:

$$f(x_1, ..., x_n) \longrightarrow f(b_1^\dagger, ..., b_n^\dagger)|\mathbf{0}\rangle, \tag{14}$$

where $|\mathbf{0}\rangle = |0\rangle^{\otimes N}$. For instance, a function $\alpha x_1^2 + \beta x_n^2$, with unknown coefficients α and β, is mapped into the state $(\alpha b_1^\dagger b_1^\dagger + \beta b_n^\dagger b_n^\dagger)|\mathbf{0}\rangle$. Thus, any general function f can be perfectly transferred as

$$\begin{aligned} &U(t_0)\; f(b_1^\dagger, ..., b_n^\dagger)\; |\mathbf{0}\rangle \\ &\quad= f(U(t_0)b_1^\dagger U^\dagger(t_0), ..., U(t_0)b_n^\dagger U^\dagger(t_0))\; |\mathbf{0}\rangle \\ &\quad= f(b_N^\dagger, ..., b_{N-n+1}^\dagger)\; |\mathbf{0}\rangle. \end{aligned} \tag{15}$$

The function f operates sequentially, from 1 to n in processor A, and reads in the opposite order in processor B. We emphasize that the central subset part of the chain (Fig. 1) should not be necessarily in the ground state and can be in an arbitrary state. However, for simplicity and without loss of generality, here, we work with the state $|\mathbf{0}\rangle$.

VI. PERFECT STATE TRANSFER WITH ON-SITE REPULSION

When the strength of H_U is relatively strong and the total number $\sum n_k$ is much smaller than the number, N, of sites, the system tends to have at most one atom at each site because of the gap caused by H_U. Moreover, in the limit when there is only one boson, we can still transfer a function perfectly. In this case, an arbitrary state $|\phi\rangle$ of the whole system can be annihilated by the on-site repulsion Hamiltonian $H_U|\phi\rangle = 0$. Therefore, this case is the same as the one in Eq. (15). The transfer would be perfect if we are able to prepare an initial state

$$|\phi\rangle_1 = \alpha|\mathbf{0}\rangle + \beta b_1^\dagger|\mathbf{0}\rangle, \tag{16}$$

where only the first site is occupied. Thus, the state can be perfectly transferred to $|\phi\rangle_N$ in the same way as in the linear case. Generally, an arbitrary state,

$$|\phi\rangle = \alpha|\mathbf{0}\rangle + \sum_{k=1}^{n}\beta_k b_k^\dagger|\mathbf{0}\rangle, \tag{17}$$

with one atom at an n-site processor A can also be transferred perfectly.

VII. GENERALIZATION

A time-independent arbitrary unitary transformation W does not change the commutation relations among angular momentum components, meaning that $H_l = \mathbf{v}\cdot\mathbf{L}$ is mapped into $H_l' = \mathbf{v}\cdot\mathbf{L}'$, where the components of $\mathbf{L}'$ satisfy the same commutation relations. However, this map may introduce new effects. Indeed, the whole family of Hamiltonians generated by an arbitrary W can transfer functions perfectly as in Eq. (13). For spin systems, this transformation W corresponds to the so-called *dressed qubit* [36]. The bosonic Hilbert space is infinite-dimensional and this allows more flexibility to use additional transformations, including continuous-variable transformations (this is not possible in

spin chains). As an example, here, we now consider transformations W such that they act on each site individually, namely, $W = \prod_{k=1}^{N} W_k$. For instance, the Hamiltonian $H_l = JL_x$ becomes

$$H_l' = H_l + \sum C_k\{2|\beta|^2 - [\beta^*(b_k + b_{k+1}) + \text{h.c.}]\} \quad (18)$$

under the displacement operator $W_k = \exp(\beta b_k^\dagger - \beta^* b_k)$. Here, we set β_k''s equal to β for all sites so that the *dressed coherent state (dressed displaced vacuum)*

$$|\boldsymbol{\beta}\rangle \equiv |\beta\rangle^{\otimes N} = \prod \exp(\beta b_k^\dagger - \beta^* b_k)|\mathbf{0}\rangle \quad (19)$$

is invariant under the transformation $U(t_0)$. Any function f is mirror transferred via the coherent state $|\boldsymbol{\beta}\rangle$:

$$f(b_1^\dagger - \beta^*, ..., b_n^\dagger - \beta^*)\ |\boldsymbol{\beta}\rangle \longrightarrow f(b_N^\dagger - \beta^*, ..., b_{N-n+1}^\dagger - \beta^*)\ |\boldsymbol{\beta}\rangle. \quad (20)$$

Another example of dressing transformation occurs via the one-mode squeezing operator $W_k = \exp[(\xi/2)(b_k^2 - b_k^{\dagger 2})]$, where the transferred Hamiltonian reads

$$H_l' = H_l \cosh\xi + H_s \sinh\xi, \quad (21)$$

where

$$H_s = \sum C_k(b_k^\dagger b_{k+1}^\dagger + b_{k+1}b_k). \quad (22)$$

This induces squeezed states. Any function f can be perfectly mirror transferred via the *dressed squeezed vacuum*

$$|\boldsymbol{\xi}\rangle \equiv |\xi\rangle^{\otimes N} = \prod \exp[(\xi/2)(b_k^2 - b_k^{\dagger 2})]|\mathbf{0}\rangle \quad (23)$$

in the same way as in Eq. (20).

We can also introduce another useful nonlinear term by setting ξ as an operator. If we bosonize $\xi = \xi_0(c + c^\dagger)$ and assume a small ξ_0, the Hamiltonian (21) becomes

$$H_l' = H_l + \xi_0(c + c^\dagger)H_s, \quad (24)$$

which describes a nonlinear down-conversion effect. If the boson ξ is coupled to a collective rotational system, $\xi = \xi_0 J_z$, the additional term in the Hamiltonian becomes $\sum C_k J_z(b_k^\dagger b_{k+1}^\dagger + b_{k+1}b_k) = H_s J_z$.

For generalizations, Refs. [3] and [12] also discussed the possibility of other Hamiltonians that might not be directly related to H_l, but play the same role as H_l. Reference [12] shows that perfect state transfers are associated with the spectra of these Hamiltonians. Here, it is useful to point out that these Hamiltonians must not have the same eigenspectra as H_l. These Hamiltonians can generate, via arbitrary dressing transformations W, other families which are not equivalent to the family of H_l.

VIII. INTERFERENCE EFFECTS DUE TO DIFFERENT NUMBERS OF SITES IN EACH PATH

The phase or signature, given by Eq. (12), with values of ± 1 and $\pm i$ forming a Z_4 cyclic group, is analogous to the phase gate for qubits. For instance, an unknown state $\alpha|\mathbf{0}\rangle + \beta\, b_1^\dagger|\mathbf{0}\rangle$ can be transferred into $\alpha|\mathbf{0}\rangle + r\,\beta\, b_N^\dagger|\mathbf{0}\rangle$. When the total number of sites, N, is even, the signature $r = \pm i$. When N is odd, $r = \pm 1$. For a given signature r, this "phase gate" is applied *during* the perfect function transfer processes.

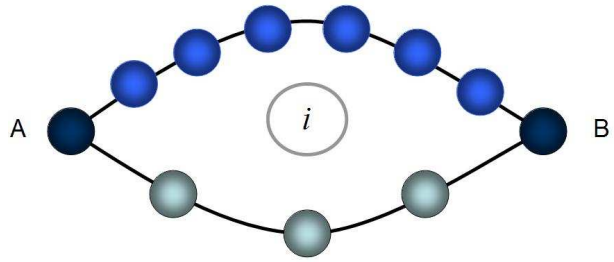

FIG. 2: (Color online) An example of site-number interference effect for two paths containing $N = 5$ sites (with signature factor $r_1 = 1$) and $N = 8$ sites (upper path with $r_2 = i$). The generalized "optical paths" in this case are the same (because both paths are of equal length), but the number of sites, N, for each path, is not. The overall phase factor r for these two paths is the product of the signature factors for each path. Thus $r = r_1 r_2 = i$. Here, $r = i$ plays a role analogous to the Aharonov-Bohm flux Φ.

This phase gate induces an important effect. Consider now site A as a sender and site B as a receiver. On a two-dimensional lattice, we can consider different paths from site A to site B. These paths can be designed such that the values of N are *different* for each path, but their "generalized optical paths" are the same. Different values of N could result in up to four different values of the signature factor r. A perfect function transfer can simultaneously occur in, for instance, two paths. This resembles the inference of two waves. States sent from site A via different paths will interfere at site B (Fig. 2). Quantum interference effects in directed paths can produce measurable effects (see, e.g., [37]).

Following Ref. [17], we can characterize the bosonic field operators

$$\psi(\mathbf{r}) = \sum_{k,p} b_k^{(p)} w(\mathbf{r} - \mathbf{r}_k^{(p)}) \quad (25)$$

at time $t = 0$, where $p = 1, 2$ denotes the two paths with the numbers of sites: N_1 and N_2. The field operator ψ evolves as:

$$\psi(\mathbf{r},t) = e^{-iJ(L_x^{(1)} + L_x^{(2)})t}\ \psi(\mathbf{r})\ e^{iJ(L_x^{(1)} + L_x^{(2)})t}, \quad (26)$$

where $L_x^{(1)}$ ($L_x^{(2)}$) is the angular momentum operator of path 1 (2). At time $t = t_0$,

$$\psi(\mathbf{r},t_0) = \sum_k [b_k^{(1)} w(\mathbf{r} - \mathbf{r}_{N_1-k+1}^{(1)}) + r\ b_k^{(2)} w(\mathbf{r} - \mathbf{r}_{N_2-k+1}^{(2)})], \quad (27)$$

where we set $r = 1$ for path 1. The average field intensity at $\mathbf{r}$ is

$$I(\mathbf{r},t) = \left\langle \psi^\dagger(\mathbf{r},t)\ \psi(\mathbf{r},t) \right\rangle, \quad (28)$$

where $\langle ... \rangle$ denotes the expectation value for the initial state. For simplicity, let us consider the processor A to be localized at the origin $\mathbf{r} = 0$. This implies that $\left\langle (b_{k'}^{(p')})^\dagger b_k^{(p)} \right\rangle = 0$, except for $k = k' = 1$. The initial intensity is

$$I(\mathbf{r}, 0) = 4 \left\langle (b_1^{(1)})^\dagger b_1^{(1)} \right\rangle |w(\mathbf{r} - \mathbf{r}_1)|^2, \qquad (29)$$

where $\mathbf{r}_1 \equiv \mathbf{r}_1^{(1)} = \mathbf{r}_1^{(2)}$. Since we start from the first site $\mathbf{r} = 0$ at time $t = 0$, we approximately set $b_1^{(1)} = b_1^{(2)}$ being the same boson. The final intensity then becomes

$$I(\mathbf{r}, t_0) = \left\langle (b_1^{(1)})^\dagger b_1^{(1)} \right\rangle |w(\mathbf{r} - \mathbf{r}_N)|^2 \, (2 + r + r^*) \quad (30)$$

because we set the two paths ending up at the same site $\mathbf{r}_N \equiv \mathbf{r}_{N_1}^{(1)} = \mathbf{r}_{N_2}^{(2)}$. The intensity I varies with different values of the signature r. For example, the intensity I in Fig. 2 will be half of $I(\mathbf{r}, 0)$. When the signature $r = +1, -1, \pm i$, the factor $(2 + r + r^*)$ in Eq. (30) is equal to 4, 0 and 2, respectively. This corresponds to constructive, destructive, and in-between interferences. This site-number-dependent interference can be generalized to include contributions from many directed paths [37]. Note that in quantum interference path-integral calculations [37], summing over directed paths can require summing over an enormously large number of different terms, instead of just four different types of terms here. In other words, the Z_4 group cyclic nature of the signature factor reduces the huge number of different terms used for standard interference calculations [37] to just summing over four possible values of r, with $r = \pm 1, \pm i$.

IX. CONCLUSIONS

We have shown, by generalizing the recent results of [9] and [10], that arbitrary functions can be sent perfectly (without state initialization and remote collaboration) through engineered interacting bosonic and qubit chains. As an example, we have analyzed perfect state transfers (using, e.g., ultracold bosonic atoms in optical lattices or polaritons in coupled cavities) described by the Bose-Hubbard model with properly designed site-dependent tunneling amplitudes (the so-called Krawtchouk lattices).

In a more general case, we have studied a family of linear and nonlinear Hamiltonians that enable perfect state transfers according to dressing transformations leading to, e.g., dressed qubits, dressed coherent state (dressed displaced vacuum), or dressed squeezed vacuum. We have also shown that one can observe quantum interference of states, transmitted in two-dimensional lattices through various paths differing solely in the numbers of intermediate sites of each path. This interference effect is analogous to the signature-dependent effects in nuclear-structure theories [16].

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